From the *pn* junction to the UFSD design

The role of numerical simulation

29.05.2018

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Summary

I. Overview of semiconductor devices
   - The *pn* junction
   - Low Gain Avalanche Detectors (LGAD)

II. Electronic device modeling
   - Analytical description
   - Numerical implementation

III. LGAD design using numerical simulations

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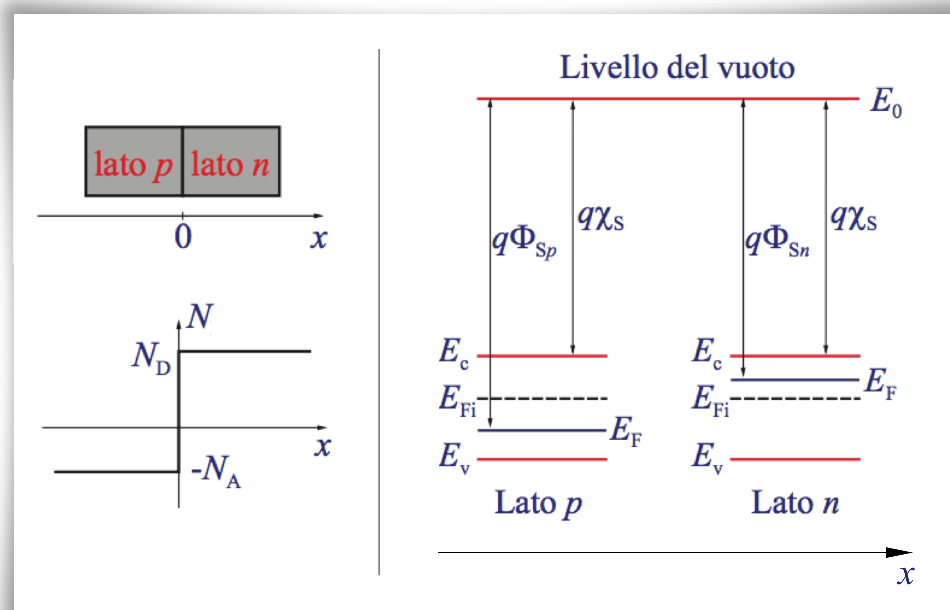
Characteristic equations of the *pn* junction

- **Definition:** the *pn* junction is a semiconductor region where a *p*-type and an *n*-type doped materials are placed side by side.
- **Example:** the *abrupt junction* of two *uniformly doped* semiconductors.



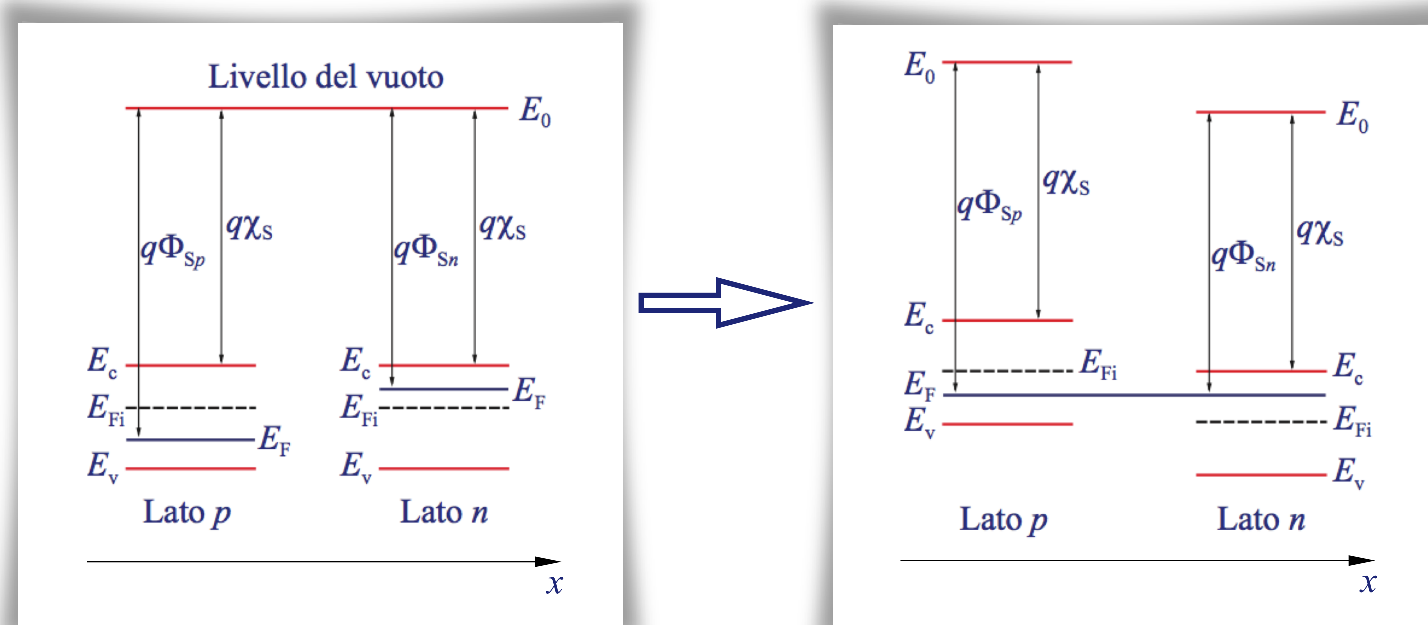
$q\chi$ is the electronic affinity (~4.05 eV in Si)
$q\Phi$ is the semiconductor work function

$$q\Phi_{Sp} = q\chi_S + E_g - (E_F - E_v) = q\chi_S + E_g - k_B T \ln \frac{N_v}{N_A}$$

$$q\Phi_{Sn} = q\chi_S + (E_c - E_F) = q\chi_S + k_B T \ln \frac{N_c}{N_D}$$

Characteristic equations of the *pn* junction

- **Golden-rules to compute the final band-diagram:**
  1. $\boldsymbol{E_g}$ and $\boldsymbol{q\chi}$ are **conserved** by definition;
  2. $\boldsymbol{E_F}$ must be **constant** across the junction;



  3. $\boldsymbol{E_0}$ and bands must be **continuous** functions (in space x).

Characteristic equations of the *pn* junction

- **How to fulfill requirement no.3 while still matching 1 and 2?**

  By introducing the concept of **space-charge region**.

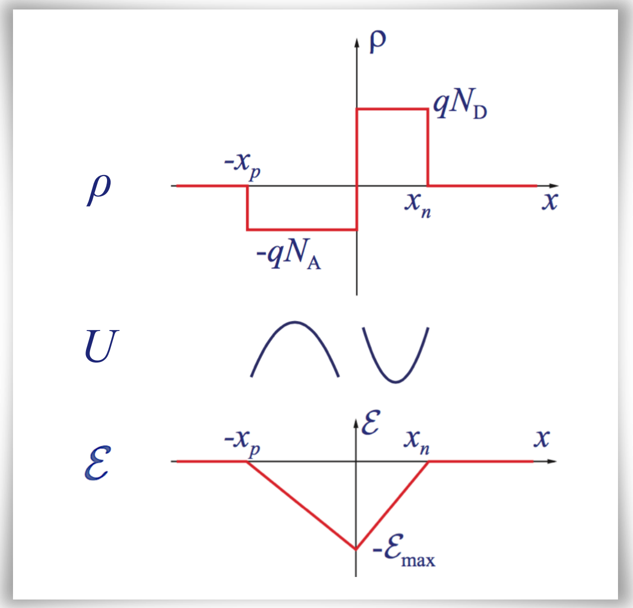
- *Observation #1*: having a **constant E_F** produces a **transient**, in which **electrons travel from the *n*-side to the *p*-side** (the vice-versa holds for holes). This mechanism behaves as a **diffusion-like dynamics**.
- *Observation #2*: the **diffusion** of *free* charges **depletes a zone** across the junction, called **space-charge region**.
- *Observation #3*: both in the ***n*- and *p*-side** of the junction *free* charges are compensated by *fixed* charges of ionized dopants (**neutral regions**, $\rho=0$), whereas in the space-charge region this is no more true. So, there, $\rho\neq0$.

Characteristic equations of the *pn* junction

- **Within the space-charge region** ($\rho \neq 0$) **the field is not a constant** and **bands are no more straight lines**. In particular, due to the **Poisson equation** of semiconductors

$$\frac{\mathrm{d}^2 U}{\mathrm{d}x^2} = q\frac{\rho}{\epsilon}$$

where $U = -q\varphi$ is the *potential energy* felt by free charges, we have



by definition of space-charge region

directly from the Poisson eq.

by 1st x-integration of the Poisson eq.:

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}x} = \frac{\rho}{\epsilon}$$

Characteristic equations of the *pn* junction

- Now consider that, **in neutral regions**,

$$\rho = 0 \Longrightarrow \frac{\mathrm{d}\mathcal{E}}{\mathrm{d}x} = 0 \Longrightarrow \mathcal{E}(x) = \mathcal{E}_0$$

so that **the *electrostatic potential***

$$\varphi(x) = -\mathcal{E}_0 x + \varphi_0$$

where $\mathcal{E} = -\frac{\partial \varphi}{\partial x}$, **is a straight line**.

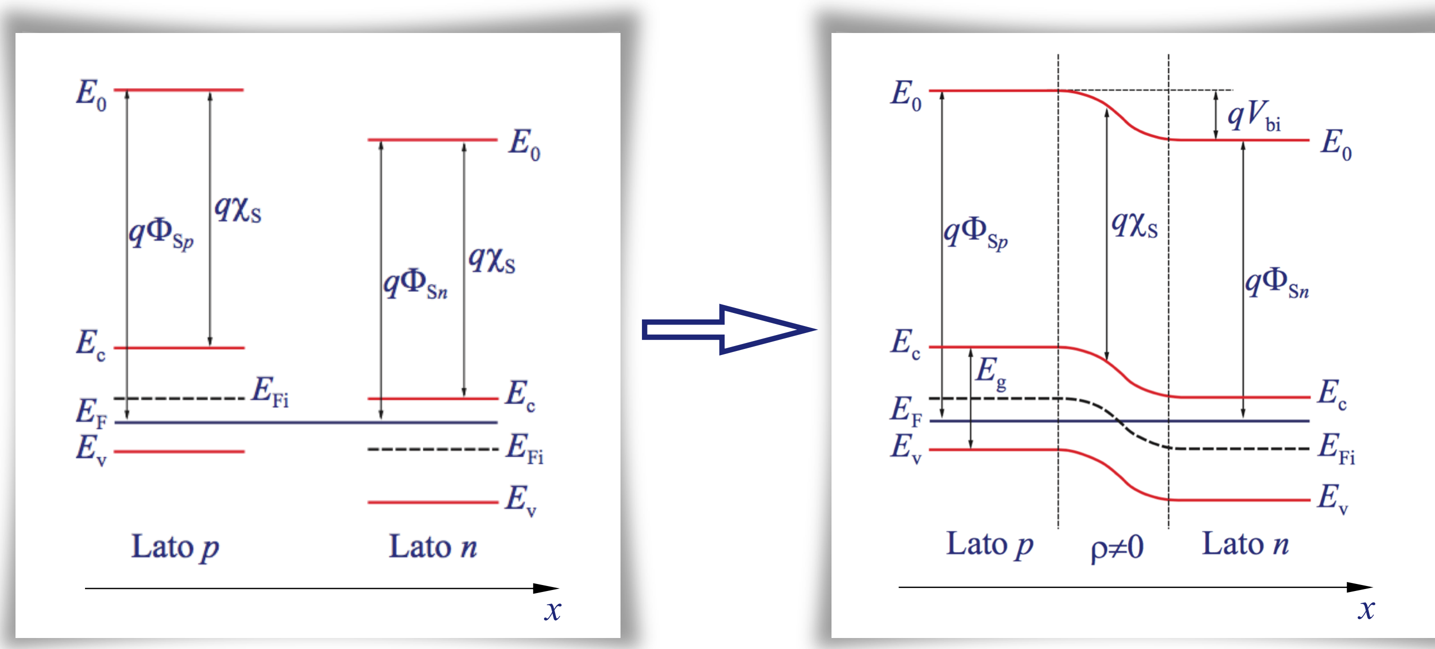
- If the junction is **at equilibrium** (without any applied bias), then

$$\mathcal{E}_0 = 0$$

and **bands are flat**.

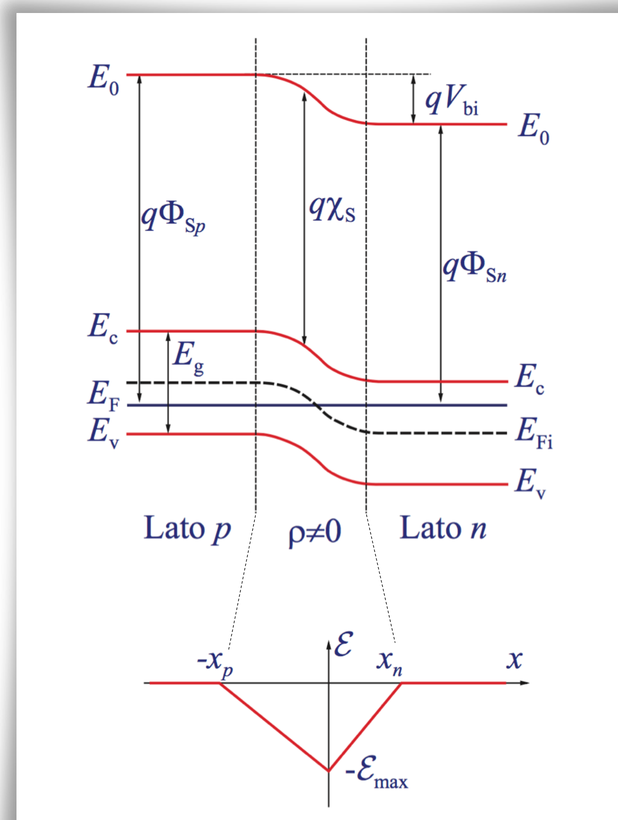
Characteristic equations of the *pn* junction

- So, finally we have:



Characteristic equations of the *pn* junction

What we concluded has several important physical implications:

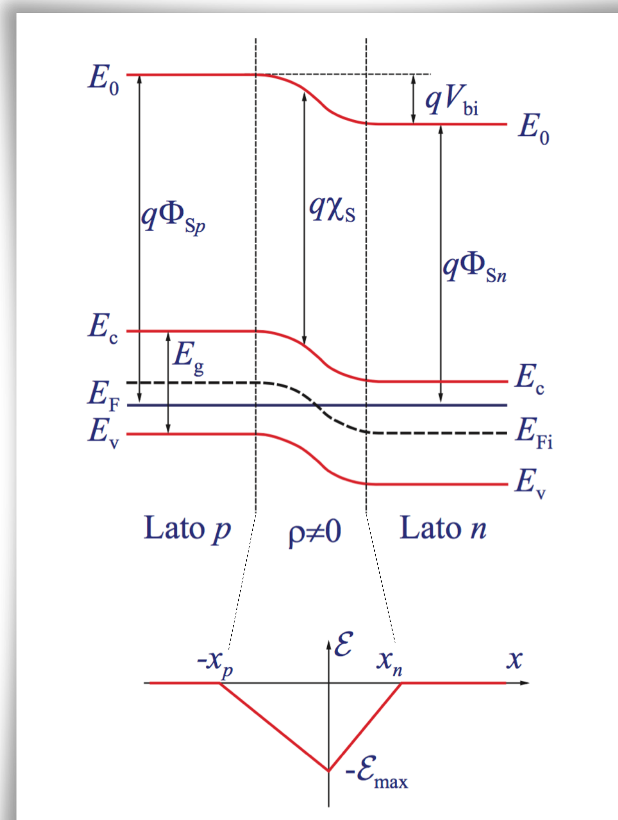


1. $\mathcal{E} \neq 0$ implies the onset of a **drift current** of carriers tending to **compensate the diffusion** of free charges such that $J = 0$;
2. A **built-in potential** qV_{bi}, created across the junction, represents an additional **barrier for the diffusion** of electrons towards the *p*-side (and holes in the *n*-side)

$$qV_{bi} = q\Phi_{Sp} - q\Phi_{Sn} = E_g - k_B T \ln \frac{N_v N_c}{N_A N_D}$$
$$= k_B T \ln \frac{N_v N_c}{n_i^2} - k_B T \ln \frac{N_v N_c}{N_A N_D}$$
$$= k_B T \ln \frac{N_A N_D}{n_i^2}$$

Characteristic equations of the *pn* junction

What we concluded has several important physical implications:



3. By integrating the Poisson equation, and thanks to the *neutrality law* $N_A x_p = N_D x_n$, one has

$$\mathcal{E}(x) = \begin{cases} -\frac{qN_A}{\epsilon}(x + x_p) & -x_p \leq x < 0 \\ \frac{qN_D}{\epsilon}(x - x_n) & 0 \leq x < x_n \end{cases}$$

and

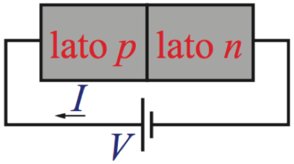
$$\mathcal{E}_{max} = \frac{qN_A}{\epsilon}x_p = \frac{qN_D}{\epsilon}x_n$$

4. In the same way:

$$\varphi(x) = \begin{cases} \frac{qN_A}{2\epsilon}(x + x_p)^2 & -x_p \leq x < 0 \\ -\frac{qN_D}{2\epsilon}(x - x_n)^2 + \frac{qN_A}{2\epsilon}x_p^2 + \frac{qN_D}{2\epsilon}x_n^2 & 0 \leq x < x_n \end{cases}$$

Characteristic equations of the *pn* junction

➤ **What happens if the junction is no more at equilibrium?**



Direct polarization

- $V > 0$, $I > 0$
- J_{diff} dominates
- electrons from *n*- to *p*-side
- holes from *p*- to *n*-side
- $qV < qV_{\mathrm{bi}}$

Reverse polarization

- $V < 0$, $I < 0$
- J_{drift} dominates
- electrons from *p*- to *n*-side
- holes from *n*- to *p*-side
- $qV > qV_{\mathrm{bi}}$

Characteristic equations of the *pn* junction

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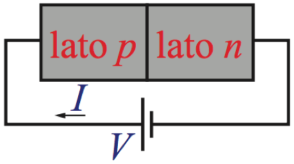
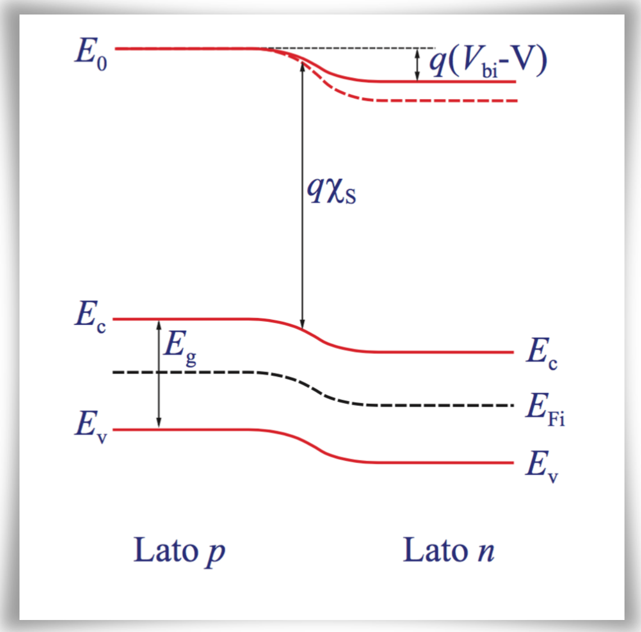
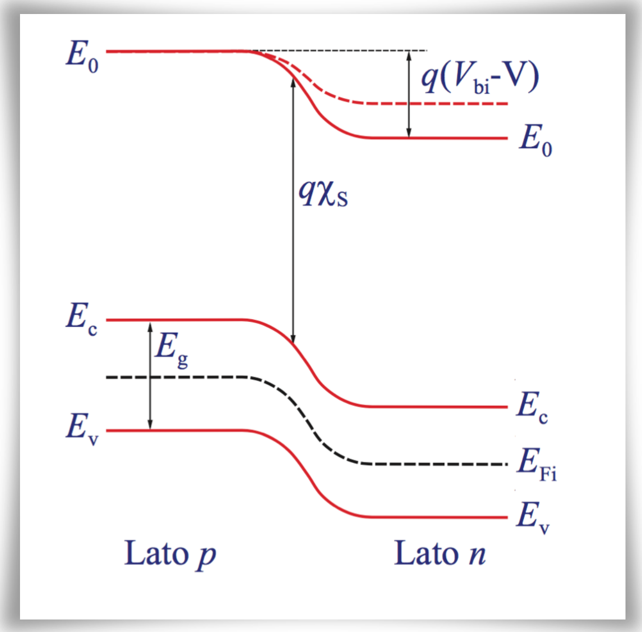


Direct polarization



Reverse polarization



Characteristic equations of the *pn* junction

Let's now deduce the static ***I*(*V*) characteristic** of the diode for a 1D structure where $I = A\,J$ (with A its cross-section) is constant at each point x.

- *Hypothesis #1*: we assume a **null field** and **quasi-equilibrium free charges** in the **neutral regions** of our system. This means that we can **neglect the drift component** of the current for minority charges:

$$
\begin{aligned}
J &\approx J_{n,\text{diff}}(x) + J_p(x) \qquad && x < -x_p \\
J &\approx J_n(x) + J_{p,\text{diff}}(x) \qquad && x > x_n
\end{aligned}
$$

- *Hypothesis #2*: **neutral regions** are **much longer than the diffusion length** $L_{n,p}$ (10s to 100s of μm) of minority charge carriers.

Being the **diffusion current** driven by the **charge density**, we have to evaluate the **excess densities** $n'(x)$ and $p'(x)$ along the whole structure.

Characteristic equations of the *pn* junction

In fact:

$$J_{n,\mathrm{diff}} = qD_n \frac{\partial n'}{\partial x}$$

$$J_{p,\mathrm{diff}} = -qD_p \frac{\partial p'}{\partial x}$$

?

with

$$D_n = \frac{kT}{q}\mu_n = V_T\,\mu_n \qquad (\leq 36\ \mathrm{cm^2/s})$$

$$D_p = \frac{kT}{q}\mu_p = V_T\,\mu_p \qquad (\leq 12\ \mathrm{cm^2/s})$$

the so-called **Einstein's diffusion coefficients**, and μ the electron/hole mobility.

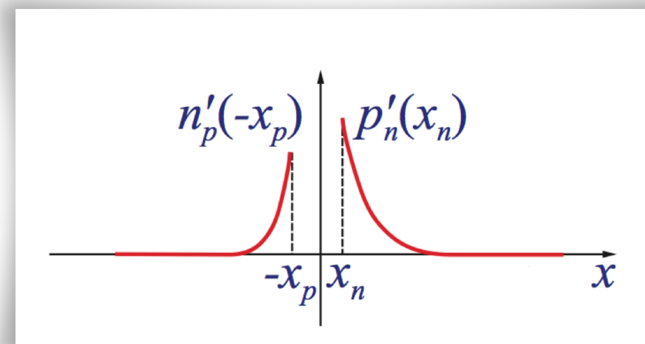
Characteristic equations of the *pn* junction

We have:

?

$$n'_p(x) = n'_p(-x_p)\exp\left(\frac{x+x_p}{L_n}\right)$$

$$p'_n(x) = p'_n(\;x_n)\exp\left(-\frac{x-x_n}{L_p}\right)$$



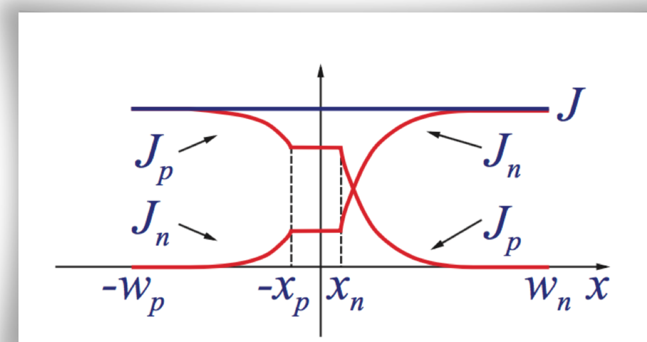
and:

$$J = J_{n,\text{diff}}(-x_p) + J_p(-x_p) = J_n(x_n) + J_{p,\text{diff}}(x_n)$$

$$J_n(x_n) \approx J_{n,\text{diff}}(-x_p) \qquad J_p(-x_p) \approx J_{p,\text{diff}}(x_n)$$

⇓

$$J \approx J_{n,\text{diff}}(-x_p) + J_{p,\text{diff}}(x_n)$$



Characteristic equations of the *pn* junction

Remembering that

$$n'_p = n_p - n_{p0} \qquad p'_n = p_n - p_{n0}\,,$$

with

$$n_{n0}(x_n) = N_\mathrm{D} \qquad n_{p0}(-x_p) = n_\mathrm{i}^2/N_\mathrm{A} \qquad p_{p0}(-x_p) = N_\mathrm{A} \qquad p_{n0}(x_n) = n_\mathrm{i}^2/N_\mathrm{D}$$

then

$$V_\mathrm{bi} = V_T \log \frac{N_\mathrm{A} N_\mathrm{D}}{n_\mathrm{i}^2} = V_T \log \frac{n_{n0}(x_n)}{n_{p0}(-x_p)} = V_T \log \frac{p_{p0}(-x_p)}{p_{n0}(x_n)}$$

$$\frac{p_{n0}(x_n)}{p_{p0}(-x_p)} = \frac{n_{p0}(-x_p)}{n_{n0}(x_n)} = \exp\left(-\frac{V_\mathrm{bi}}{V_T}\right)$$

Characteristic equations of the *pn* junction

Rewriting

$$\frac{p_{n0}(x_n)}{p_{p0}(-x_p)} = \frac{n_{p0}(-x_p)}{n_{n0}(x_n)} = \exp\left(-\frac{V_{\mathrm{bi}}}{V_T}\right)$$

in the *low-injection regime*

$$\frac{p_n(x_n)}{p_p(-x_p)} \approx \frac{n_p(-x_p)}{n_n(x_n)} \approx \exp\left(-\frac{V_{\mathrm{bi}} - V}{V_T}\right)$$

and assuming

$$n_n(x_n) \approx n_{n0}(x_n) = N_{\mathrm{D}} \qquad p_p(-x_p) \approx p_{p0}(-x_p) = N_{\mathrm{A}}$$

we have the **junction law**

$$n_p(-x_p) = n_{p0}(-x_p)\exp\left(\frac{V}{V_T}\right) \qquad p_n(x_n) = p_{n0}(x_n)\exp\left(\frac{V}{V_T}\right)$$

so that

$$\frac{n'_p(-x_p)}{n_{p0}(-x_p)} = \exp\left(\frac{V}{V_T}\right) - 1 \qquad \frac{p'_n(x_n)}{p_{n0}(x_n)} = \exp\left(\frac{V}{V_T}\right) - 1$$

Characteristic equations of the *pn* junction

Now, by plugging the junction law into the current densities

$$p_n(x_n) = \frac{n_\mathrm{i}^2}{N_\mathrm{D}} \exp\left(\frac{V}{V_T}\right) \qquad J_{n,\mathrm{diff}} = qD_n \frac{\partial n'}{\partial x}$$

$$n_p(-x_p) = \frac{n_\mathrm{i}^2}{N_\mathrm{A}} \exp\left(\frac{V}{V_T}\right) \qquad J_{p,\mathrm{diff}} = -qD_p \frac{\partial p'}{\partial x}$$

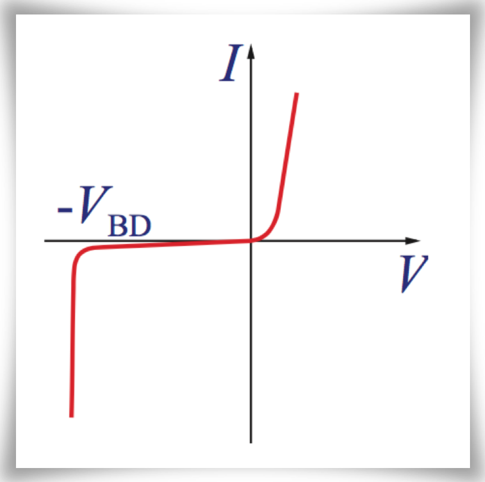
one finds

$$J_{n,\mathrm{diff}}(-x_p) = qD_n \frac{n_p'(-x_p)}{L_n}$$

$$J_{p,\mathrm{diff}}(x_n) = qD_p \frac{p_n'(x_n)}{L_p}$$

and we end up with the expression

$$I = qA\frac{n_\mathrm{i}^2}{N_\mathrm{A}}\frac{D_n}{L_n} + qA\frac{n_\mathrm{i}^2}{N_\mathrm{D}}\frac{D_p}{L_p}\left[\exp\left(\frac{V}{V_T}\right) - 1\right]$$



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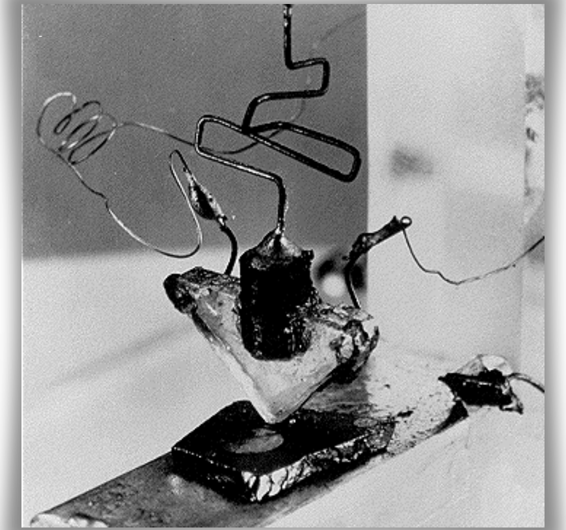
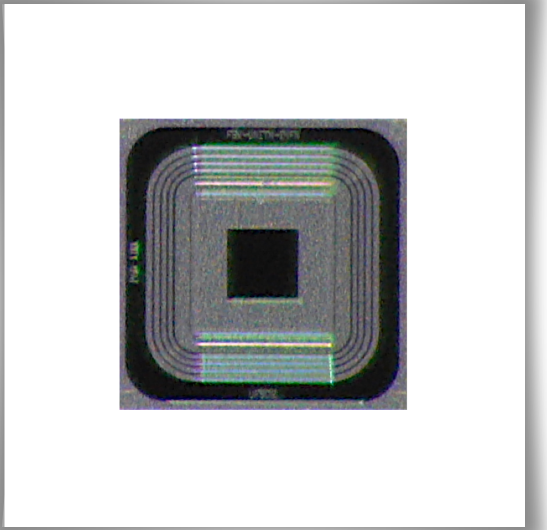
Towards a technological step…

first *n*-*p*-*n* "tip"-transistor



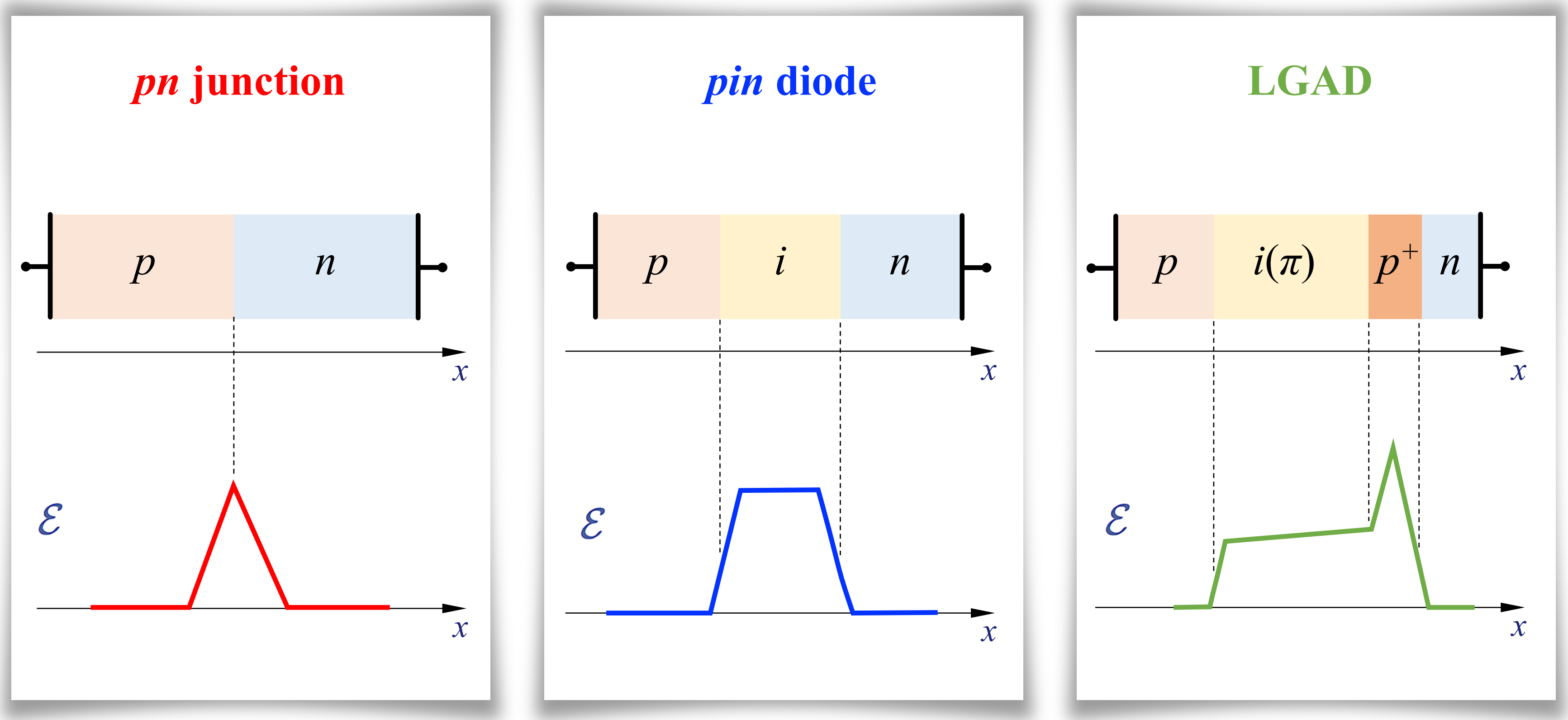
J. Bardeen, W. Brattain, W. Shockley
Bell Labs. - NJ
(1948)

Ultra Fast Silicon Detector



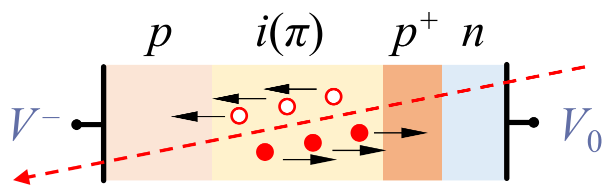
UFSD Group
INFN Torino and FBK Trento
(2018)

Extending our application domain to other systems



Working principles of an LGAD

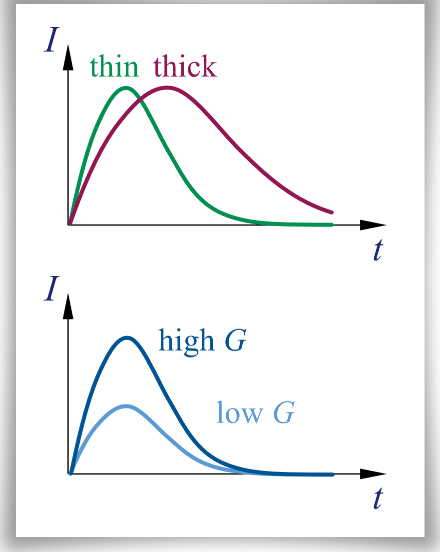
➤ **What is charge multiplication in LGAD?**



- **Primary charges** (electron/hole pairs) are produced **by ionization**, while the particle is crossing the sensor;
- Due to the **reverse field**, electrons **drift** towards the *n*-side and holes towards the *p*-side;
- When electrons travel along the $\boldsymbol{p^+}$ **region** (the *gain*- or *multiplication*-layer) they experience an **high field**;
- This field is responsible for the **impact ionization**, which produces an avalanche **multiplication of secondary charges**;
- Now the **total current** is due to the **additional avalanche contribution**.

Working principles of an LGAD

➤ **Why using LGAD to detect particles at CERN?**



I. We need **charge multiplication**:

1. LGAD exploit the so-called **avalanche multiplication**, a process which belongs to the class of **generation/recombination (GR) mechanisms**;
2. Charge multiplication allows to obtain **large and fast signals**:
   - the **thinner** the sensor, the **faster** the signal;
   - the **higher** the **gain**, the **larger** the signal.

II. We need a good **S/N**:

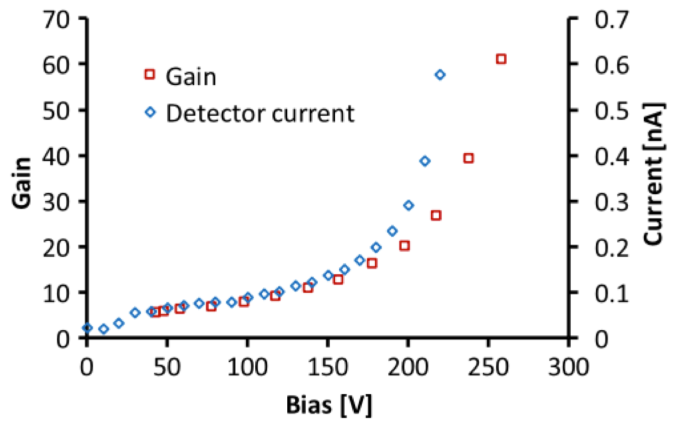
1. Also the **noise** related to the current signal is **proportional to the gain**
2. The **gain** ***G*** has to be kept as **low** as required by electronics ($G \sim$ 10-20)

Working principles of an LGAD

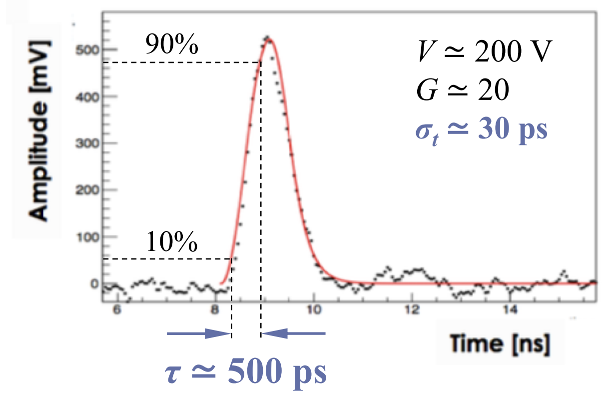
➤ Why using LGAD to detect particles at CERN?

Examples of 50 μm LGAD performance:

reverse current & gain versus bias V



current signal (test beam @ CERN-H8)



Working principles of an LGAD

- **Can we predict the avalanche contribution to the total current?**

Let's introduce a bit of physical-mathematics…

1. The **avalanche** process is modeled via its **ionization coefficient** α, i.e. the **inverse of the electron/hole mean free path** (cm^{-1});

2. In the literature, several **expressions of** $\boldsymbol{\alpha}$ are available. In general, all of them are based on the **Chynoweth's theory** (1958), according to which:

$$\alpha_{n,p}(\mathcal{E}) = \gamma\, A_{n,p}\, \exp\left(-\gamma \frac{B_{n,p}}{\mathcal{E}}\right)$$

3. Once the coefficient has been obtained, one has to evaluate the **net avalanche generation rate** U_{aval}, i.e. the **number of multiplied** $\mathbf{e^-/h^+}$ **pairs per volume** (cm^{-3}) **per unit time** (s^{-1}), as:

$$U_{\mathrm{aval}} = \frac{\mathrm{d}n}{\mathrm{d}t} = \frac{\mathrm{d}p}{\mathrm{d}t} = \alpha_n n v_n + \alpha_p p v_p$$

… Now we need a complete description of the system!

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Electronic device modeling - I

- We recall the **twofold nature of the current** in a semiconductor device:
  - *a.* **Drift current**, driven by the *electric field*;
  - *b.* **Diffusion current**, due to the density gradient of *free charges*.

$$J_{n,\text{dr}} = qn\mu_n\mathcal{E} \qquad J_{p,\text{dr}} = qp\mu_p\mathcal{E}$$

$$J_{n,\text{diff}} = qD_n\frac{\partial n}{\partial x} \qquad J_{p,\text{diff}} = -qD_p\frac{\partial p}{\partial x}$$

$$J = \underbrace{J_{n,\text{diff}} + J_{n,\text{dr}}}_{J_n} + \underbrace{J_{p,\text{diff}} + J_{p,\text{dr}}}_{J_p}$$

- Then we introduce (all) the **GR mechanisms** through their **net rates *U***:

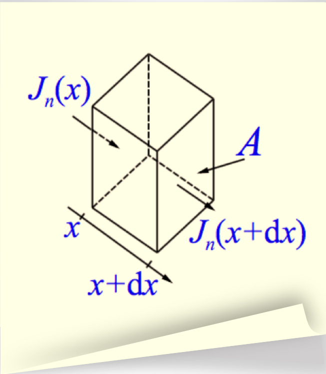
$$U_n = R_n - G_n \approx \frac{n - n_0}{\tau_n} = \frac{n'}{\tau_n} \qquad U_p = R_p - G_p \approx \frac{p - p_0}{\tau_p} = \frac{p'}{\tau_p}$$

Electronic device modeling - I

- To derive the **global current density** (field + charge + GR):
  1. in a volume $\mathrm{d}V = A\mathrm{d}x$ the **time variation of the electron density** (similarly for holes) is

field + charge — GR

$$\frac{\partial n}{\partial t}A\mathrm{d}x = \frac{J_n(x)}{-q}A - \frac{J_n(x+\mathrm{d}x)}{-q}A + G_n A\mathrm{d}x - R_n A\mathrm{d}x$$



  2. by using the 1st-order Taylor series expansion

$$J_n(x+\mathrm{d}x) \approx J_n(x) + \frac{\partial J_n}{\partial x}\mathrm{d}x$$

and assuming $\mathrm{d}x \longrightarrow 0$, we obtain the **continuity equations**:

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - U_n \qquad \frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - U_p$$

Electronic device modeling - I

- Since the drift component depends on the **electric field**, we need a third equation to close the system, the **Poisson equation**, which connects the *field* to the *charge densities*.
  The final (1D) mathematical framework is:

continuity eqs. →

$$\begin{cases} \dfrac{\partial n}{\partial t} = \dfrac{1}{q}\dfrac{\partial J_n}{\partial x} - U_n \\ \dfrac{\partial p}{\partial t} = -\dfrac{1}{q}\dfrac{\partial J_p}{\partial x} - U_p \\ \dfrac{\partial \mathcal{E}}{\partial x} = \dfrac{\rho}{\epsilon} \end{cases}$$

Poisson eq. →

DRIFT-DIFFUSION MODEL (DD)

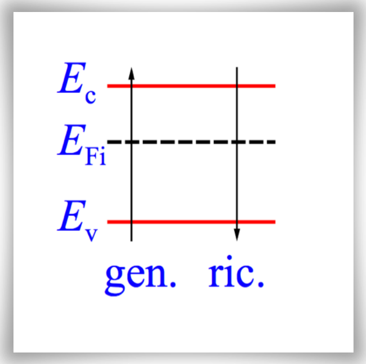
where $J_n = q\mu_n n\mathcal{E} + qD_n\dfrac{\partial n}{\partial x}$, $J_p = q\mu_p p\mathcal{E} - qD_p\dfrac{\partial p}{\partial x}$ **TRANSPORT EQS.**

and $\mathcal{E} = -\dfrac{\partial \varphi}{\partial x}$, $\rho = q\left(p - n + N_D^+ - N_A^-\right)$.

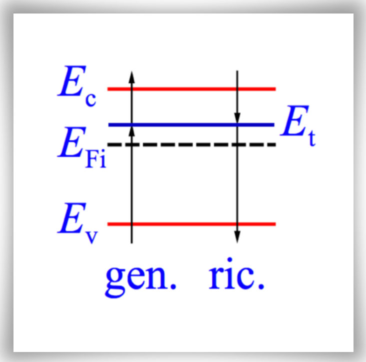
Electronic device modeling - I

- Avalanche generation is not the only **GR mechanism** occurring in silicon devices. In general, we have to account for **two different families**:

A. **Band-to-band** generation/recombination
   - *Auger*
   - *direct tunneling*
   - …



B. **Defect-assisted** generation/recombination
   - *Shockley-Read-Hall* (*SRH*)
   - *trap-assisted tunneling*
   - …



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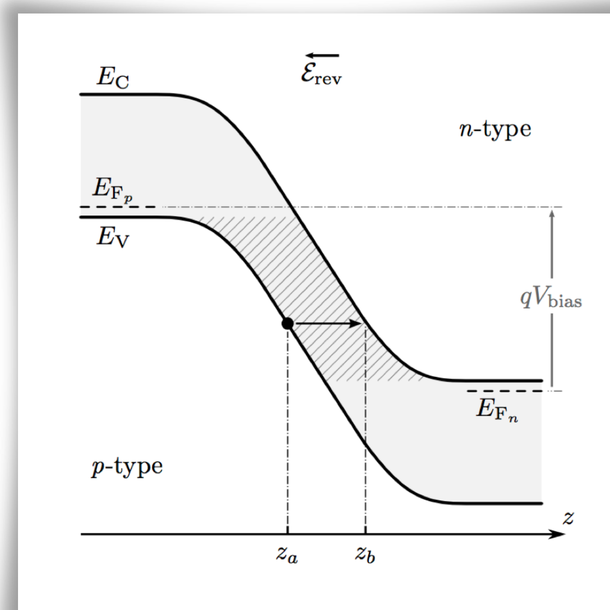

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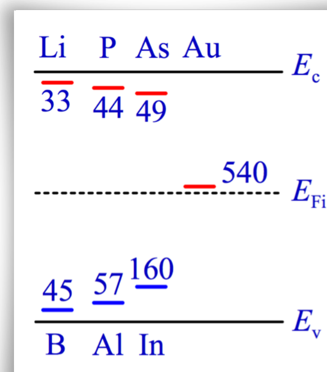
B. **Defect-assisted** generation/recombination
   - *Shockley-Read-Hall* (*SRH*)
   - *trap-assisted tunneling*
   - …



Electronic device modeling - I

- **SRH** processes are determined by such a net rate statistics

$$U_{\mathrm{SRH}} = \frac{np - n_i^2}{\tau_p \left(n + n_i \, \mathrm{e}^{\frac{E_{\mathrm{trap}} - E_{\mathrm{F}_i}}{k_{\mathrm{B}}T}} \right) + \tau_n \left(p + n_i \, \mathrm{e}^{\frac{E_{\mathrm{F}_i} - E_{\mathrm{trap}}}{k_{\mathrm{B}}T}} \right)}$$

  where $\tau_{n,p}$ are proper **electron/hole lifetimes**, i.e. the average time interval (~10^{-7}-10^{-9} s) between two consecutive scattering processes originating (or annihilating) e^-/h^+ pairs.

- Moreover, **band-to-band tunneling** is modeled with the usual Kane expression (1961)

$$U_{\mathrm{tunn}} = A\,\mathcal{E}^2 \cdot \exp\left(-B/\mathcal{E}\right)$$

  with A and B (V/cm) material-dependent parameters.

Summary

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Electronic device modeling - II

- **We need a method to compute the DD model**

$$\begin{cases} \frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - U_n \\ \frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - U_p \\ \frac{\partial \mathcal{E}}{\partial x} = \frac{\rho}{\epsilon} \end{cases}$$

where φ is the input function, n, p and $\mathcal{E}$ are the unknowns of the continuity equations and where the Poisson equation closes the system.

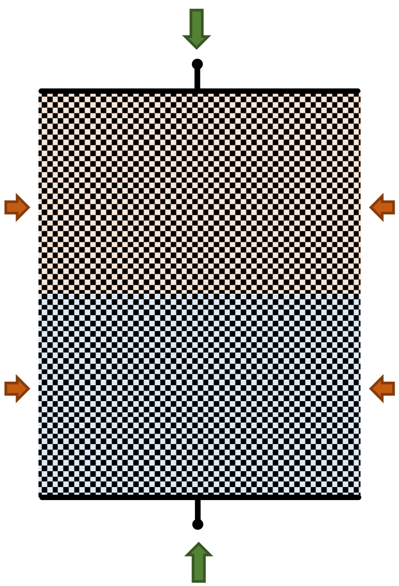
- **We have to solve a set of *non-linear*, *secondary-order* PDEs, in *space* and *time*, for the whole device!**

Electronic device modeling - II

- **The strategy**

- Dynamics (bias ramps, transients, …) is treated as a sequence of *small* increments between stationary states at equilibrium: the **quasi-stationary process**;

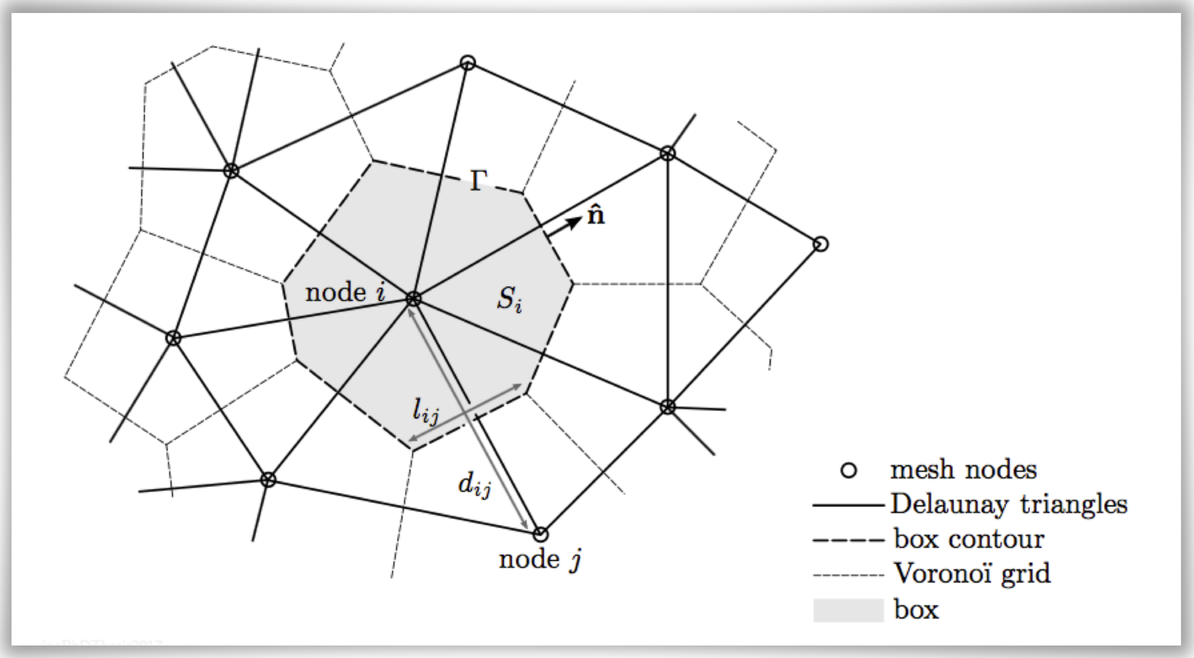
- At each quasi-stationary step the mathematics has to be simplified through proper approximations and algorithms:



1. the **geometry** is **discretized** (e.g.: Delaunay-Voronoï procedure)
2. DD system is rewritten and adapted to the mesh grid
3. PDEs are linearized and transformed into **ODEs** (**FD schemes**)
4. **I.C.** and **B.C.** are defined
5. the *new* DD model is solved via **iterative methods** (Newton) in all mesh nodes

Electronic device modeling - II

1. The **geometry** is **discretized** (e.g.: **Delaunay-Voronoï procedure**)



design of **nodes**

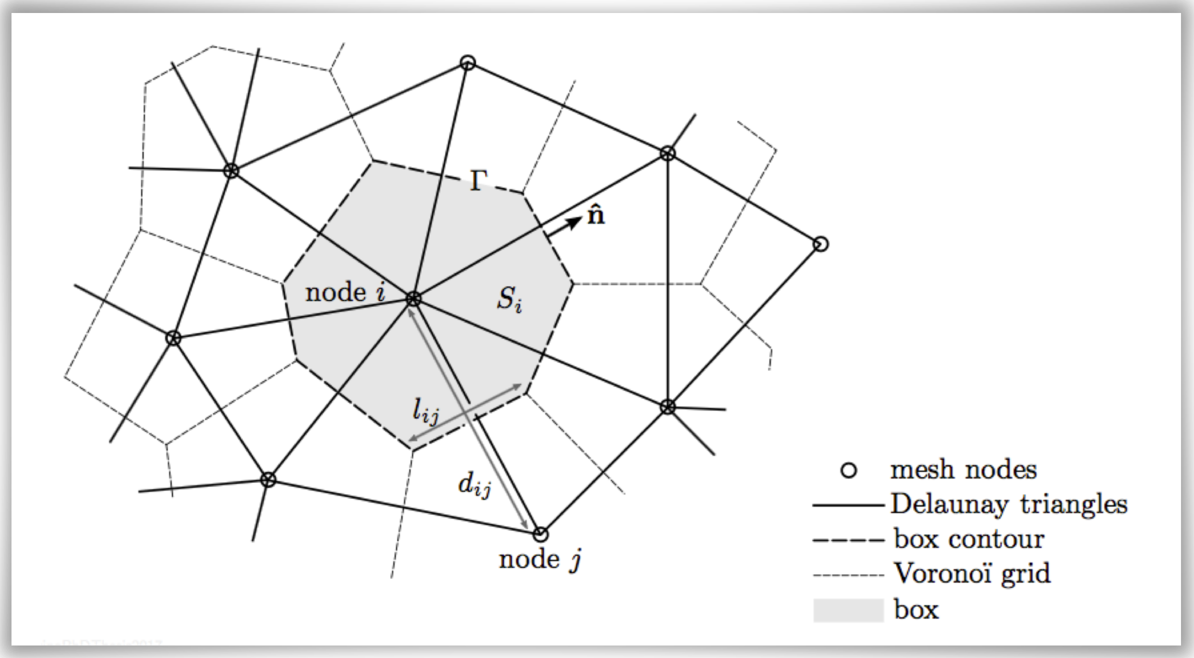
creation of **non-obtuse triangles**

creation of **boxes**

$\mathbf{J}_{\perp n,p}$ are conserved at boxes interfaces

Electronic device modeling - II

2. **Drift-Diffusion system** is rewritten and **adapted** to the **mesh grid**



scalar/vector operators and **constants** are transformed:

$$\frac{\partial}{\partial t}\int_S x\,\mathrm{d}s \Rightarrow \frac{\partial x_i}{\partial t}\,S_i$$

$$\oint_\Gamma \mathbf{F}_\perp\,\mathrm{d}\gamma \Rightarrow \sum_j l_{ij}\,\langle \mathbf{F}_\perp\rangle_{ij}$$

$$\int_S c\,\mathrm{d}s \Rightarrow c_i\,S_i$$

by **averaging** the **in/out quantities** at each box side, they are computed at **nodes**

Electronic device modeling - II

3. PDEs are linearized and transformed into **ODEs (FD schemes)**

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - U_n$$
$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - U_p$$
$$\frac{\partial \mathcal{E}}{\partial x} = \frac{\rho}{\epsilon}$$

FD central differences + Scharfetter-Gummel scheme

$$\langle \mathcal{E}_\perp \rangle_{ij} = -\frac{\partial \phi_{ij}(\mathbf{r},t)}{\partial \mathbf{r}} \approx \frac{\phi_i(\mathbf{r},t) - \phi_j(\mathbf{r},t)}{d_{ij}}$$

$$\begin{aligned} \frac{1}{q}\langle \mathbf{J}_{\perp_n} \rangle_{ij} &\approx \frac{1}{q}\nabla \mathbf{J}_{\perp_n} \\ &= -\mu_n\, n(\mathbf{r})\,\frac{\partial \phi_{ij}(\mathbf{r},t)}{\partial \mathbf{r}} + D_n \frac{\mathrm{d}n(\mathbf{r},t)}{\mathrm{d}\mathbf{r}} \\ &\approx -\mu_n\, n(\mathbf{r},t)\,\frac{\phi_i(\mathbf{r}) - \phi_j(\mathbf{r},t)}{d_{ij}} + D_n \frac{\mathrm{d}n(\mathbf{r},t)}{\mathrm{d}\mathbf{r}} \\ &\overset{\text{SG}}{\approx} \frac{D_n}{d_{ij}}\left[n_j(\mathbf{r},t)\,\mathrm{B}(\Delta_{ij}) - n_i(\mathbf{r},t)\,\mathrm{B}(-\Delta_{ij})\right] \end{aligned}$$

with:

$$\Delta_{ij} = q\,\frac{\phi_i(\mathbf{r},t) - \phi_j(\mathbf{r},t)}{k_\mathrm{B}T} = \frac{\phi_i(\mathbf{r},t) - \phi_j(\mathbf{r},t)}{V_\mathrm{T}}$$

and

$$\mathrm{B}(\alpha) = \frac{\alpha}{e^\alpha - 1}$$

$$\frac{\partial n_i(\mathbf{r},t)}{\partial t} = \sum_j \frac{D_n\, l_{ij}}{d_{ij}\, S_i}\left[n_j(\mathbf{r},t)\,\mathrm{B}(\Delta_{ij}) - n_i(\mathbf{r},t)\,\mathrm{B}(-\Delta_{ij})\right] - U_{n,i}(\mathbf{r},t)$$

$$\frac{\partial p_i(\mathbf{r},t)}{\partial t} = -\sum_j \frac{D_p\, l_{ij}}{d_{ij}\, S_i}\left[p_i(\mathbf{r},t)\,\mathrm{B}(\Delta_{ij}) - p_j(\mathbf{r},t)\,\mathrm{B}(-\Delta_{ij})\right] - U_{p,i}(\mathbf{r},t)$$

$$\begin{aligned} \frac{\partial \mathcal{E}}{\partial x} &= \sum_j l_{ij}\,\langle \mathcal{E}_\perp \rangle_{ij} \\ &\approx \sum_j l_{ij}\,\frac{\phi_i(\mathbf{r},t) - \phi_j(\mathbf{r},t)}{d_{ij}} \end{aligned}$$

Electronic device modeling - II

4. **I.C.** and **B.C.** are defined

Initial Conditions: starting polarization at contacts

Boundary Conditions:

$$\frac{\partial n(\mathbf{r},t)}{\partial \hat{\mathbf{n}}} = 0\,, \quad \frac{\partial p(\mathbf{r},t)}{\partial \hat{\mathbf{n}}} = 0 \quad \text{and} \quad \frac{\partial \phi(\mathbf{r},t)}{\partial \hat{\mathbf{n}}} = 0$$

Neumann *homogeneous* (insulators, external edges,…)

$$\begin{cases} n(\mathbf{r},t)\,\mu_n \dfrac{\partial \phi(\mathbf{r},t)}{\partial \hat{\mathbf{n}}} = D_n \dfrac{\partial n(\mathbf{r},t)}{\partial \hat{\mathbf{n}}} \\ p(\mathbf{r},t)\,\mu_p \dfrac{\partial \phi(\mathbf{r},t)}{\partial \hat{\mathbf{n}}} = -D_p \dfrac{\partial p(\mathbf{r},t)}{\partial \hat{\mathbf{n}}} \\ \epsilon_{\mathrm{s}} \left.\dfrac{\partial \phi(\mathbf{r},t)}{\partial \hat{\mathbf{n}}}\right|_{\mathrm{s}} = \epsilon_{\mathrm{diel}} \left.\dfrac{\partial \phi(\mathbf{r},t)}{\partial \hat{\mathbf{n}}}\right|_{\mathrm{diel}} \end{cases}$$

Neumann *non-homogeneous* (dielectrics)

$$\begin{cases} n(\mathbf{r},t) = \dfrac{1}{2}\left(\sqrt{\displaystyle\sum_k C_k^{\pm 2}(\mathbf{r},t) + 4n_i^2} + \displaystyle\sum_k C_k^{\pm}(\mathbf{r},t)\right) \\ p(\mathbf{r},t) = \dfrac{1}{2}\left(\sqrt{\displaystyle\sum_k C_k^{\pm 2}(\mathbf{r},t) + 4n_i^2} - \displaystyle\sum_k C_k^{\pm}(\mathbf{r},t)\right) \\ \phi(t) = V_{\mathrm{bias}}(t) + \text{const.} \end{cases}$$

Dirichlet *non-homogeneous* (contacts)

Electronic device modeling - II

5. the *new* DD model is solved via **iterative methods** (Newton) in all mesh nodes

   a. Find an **initial guess** for the potential;

   b. Choose a **maximum number of iterations** and a **tolerance** (max. difference between the solution and our guess);

   c. The **initial solution** (at equilibrium) is obtained by **solving only the Poisson equation** using the **I.C.** and **B.C.**;

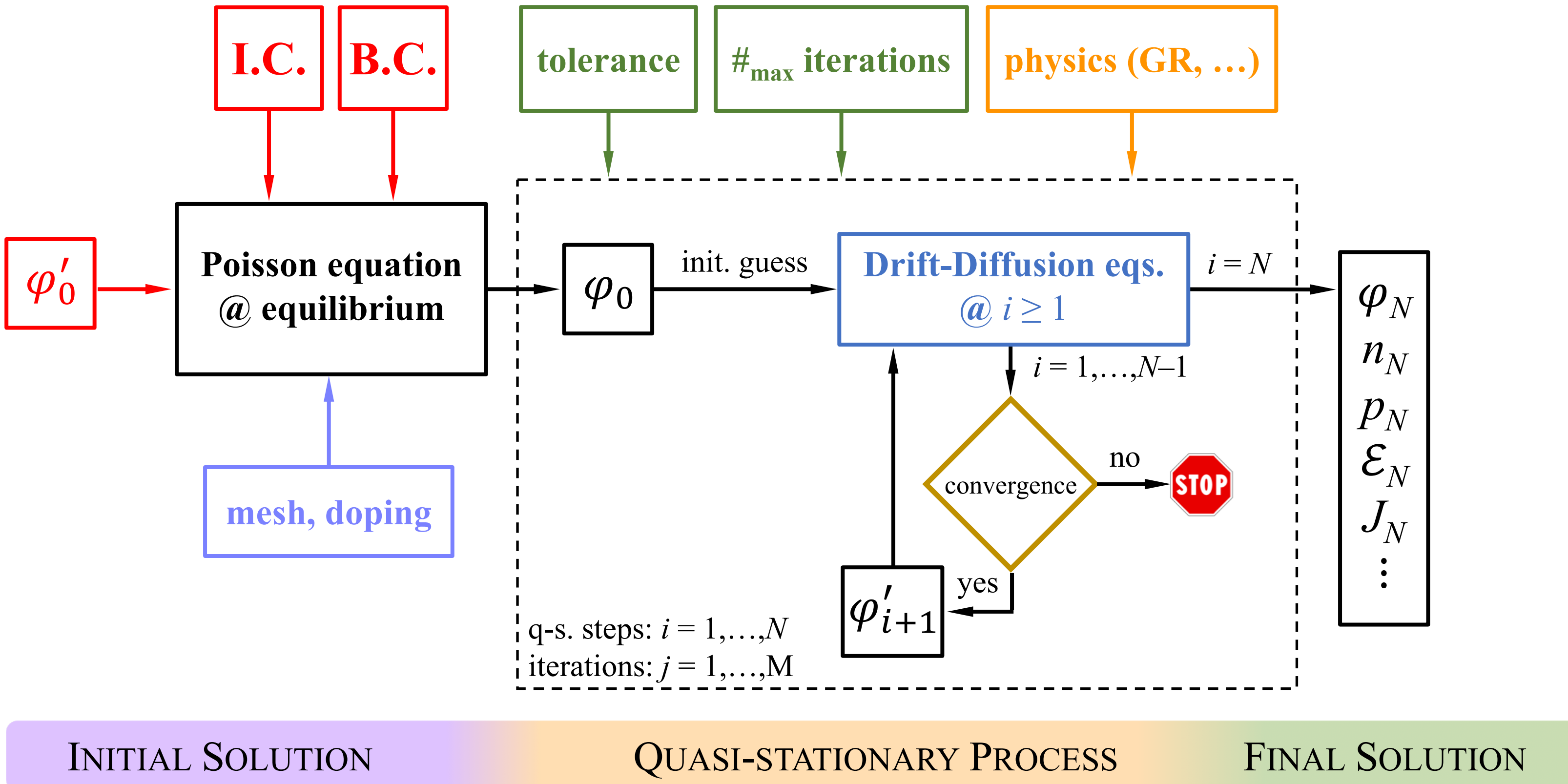
   d. If we perform a **bias ramp**, or a transient, each step of the ramp is treated as a **quasi-stationary state**. The Poisson solution is used as initial guess for solving the **continuity equations** at step $i = 1$;

   e. At each state, the **solution** is a function of the **previous two steps**, if available (*Newton-Raphson scheme*):

      - If the **solution** is found **within the maximum number of iterations** and with an **error less than the tolerance**, then **the system converges** and the scheme go further, otherwise the method is aborted;
      - The **self-consistent solution** of the **Poisson-continuity equations** (DD) at steps $i \geq 1$ proceeds with the same scheme until the end of the ramp.

Electronic device modeling - II

5. the *new* DD model is solved via **iterative methods** (Newton) in all mesh nodes



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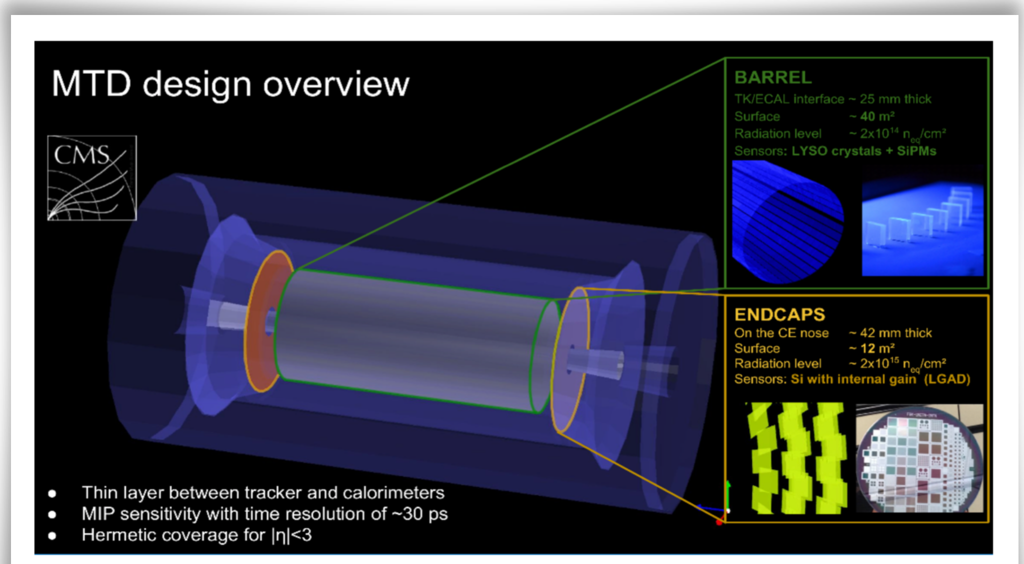
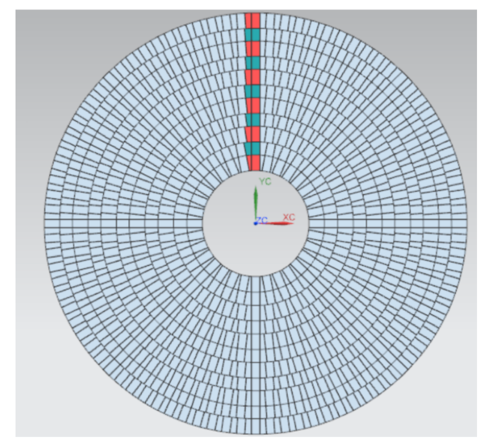
Design and simulation of LGAD

➤ Why LGAD are so innovative?

- **Large signals** coupled with **low Gain** $\Longrightarrow$ **high S/N**
- **Fast signals** $\Longrightarrow$ **high time resolution**
- **Simple design** $\Longrightarrow$ **low production cost**
- **Huge ongoing R&D** $\Longrightarrow$ **radiation hardness**

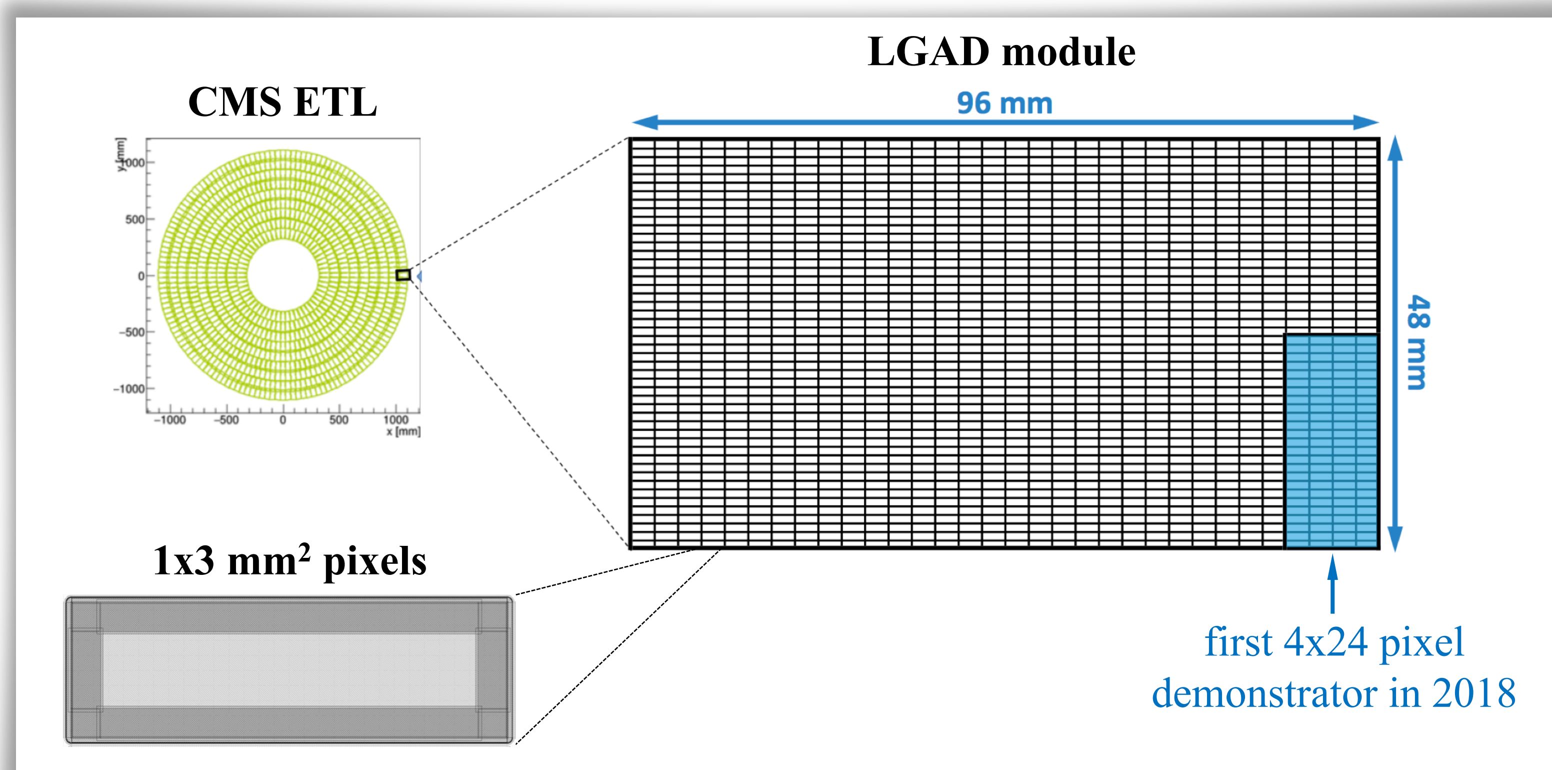


	Endcap LGAD
Coverage	$1.5 < \lvert\eta\rvert < 3.0$
Surface Area	~ 12 m^2
Power Budget	$\sim$1.8 kW/m^2
Radiation Dose	$\leq$ 2e15 neq/cm^2
Installation Date	2024



Design and simulation of LGAD

➤ **How a real LGAD module is made?**



Design and simulation of LGAD

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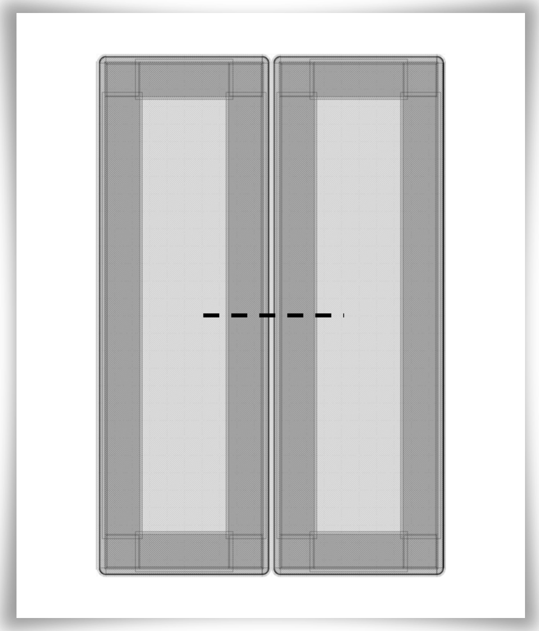
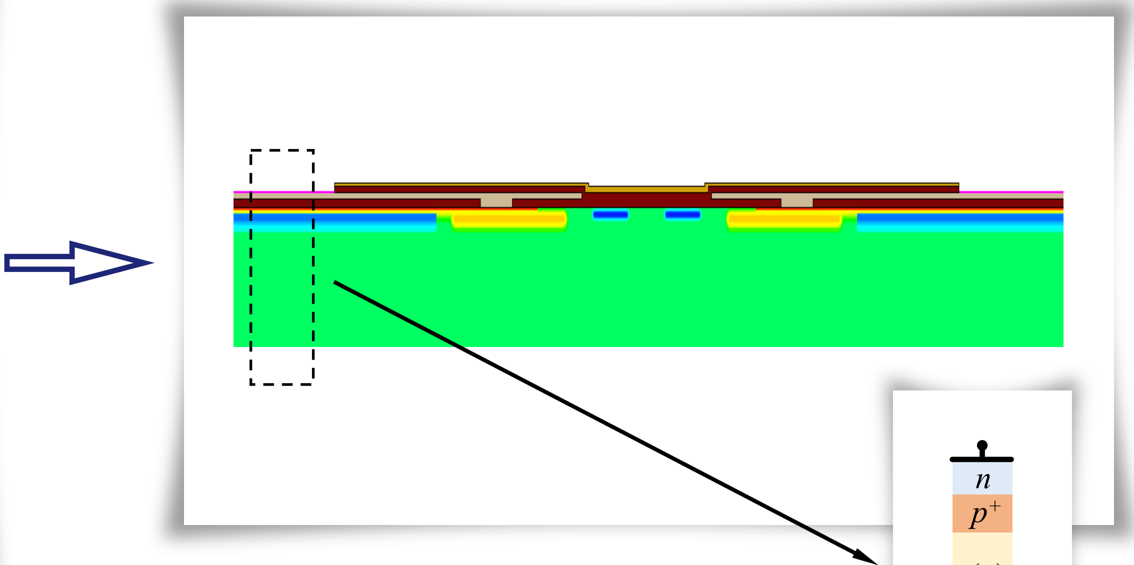
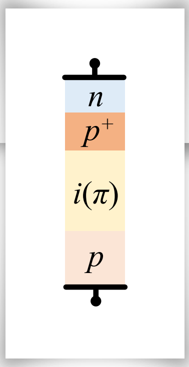




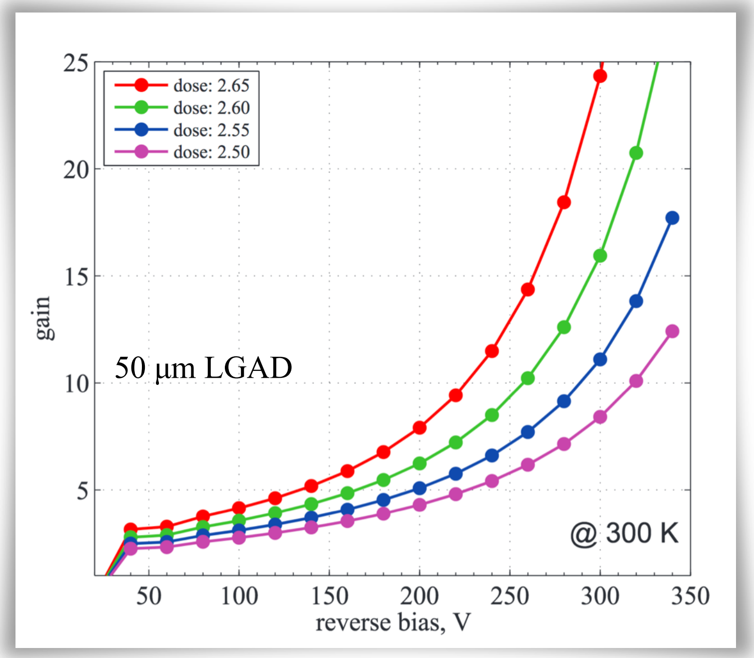


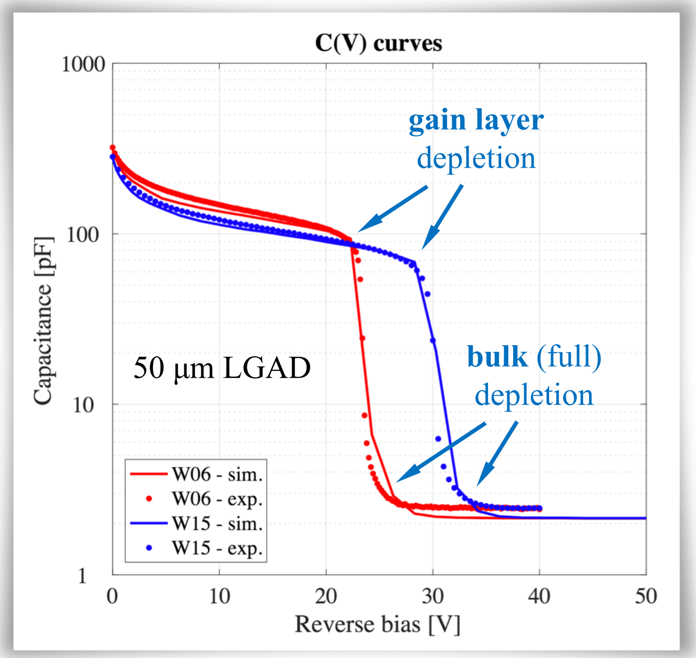
Three main R&D areas

1. Study of detector **static/dynamic characteristics**: $I(V)$, $G(V)$, $C(V)$, …
2. Analysis about the internal **electric fields**: efficiency issues, …
3. **Radiation tolerance**

Design and simulation of LGAD

1. Study of detector **static/dynamic characteristics**: $I(V)$, $G(V)$, $C(V)$, …

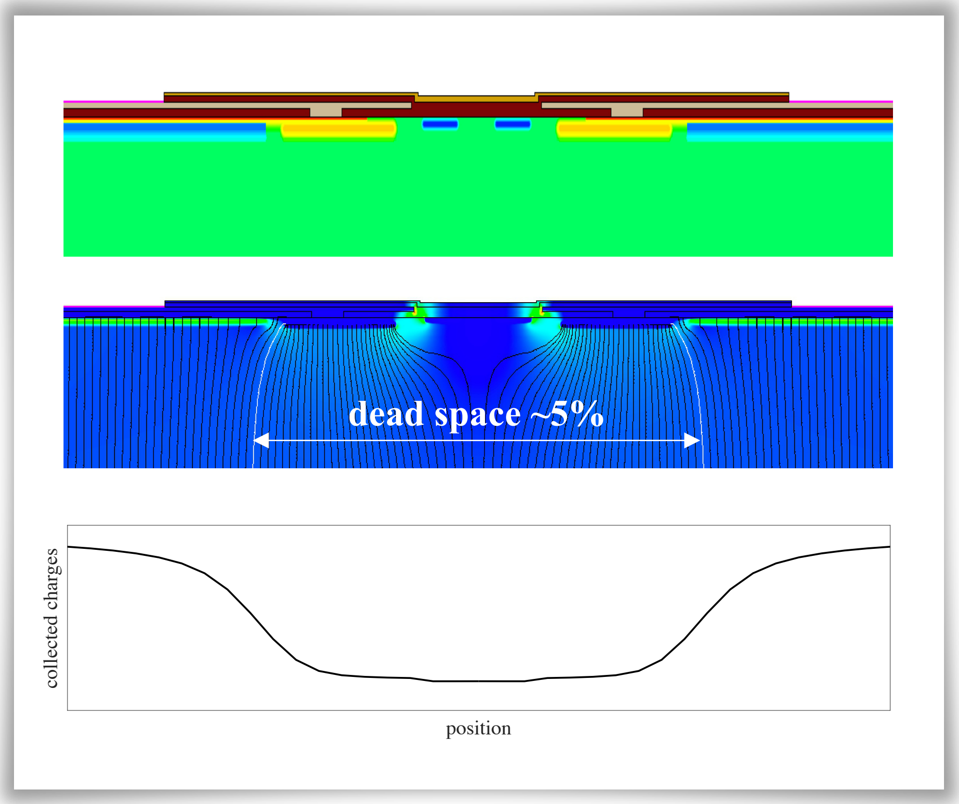




$$\alpha_{n,p}(\mathcal{E}) = \gamma\, A_{n,p} \exp\left(-\gamma \frac{B_{n,p}}{\mathcal{E}}\right)$$

Design and simulation of LGAD

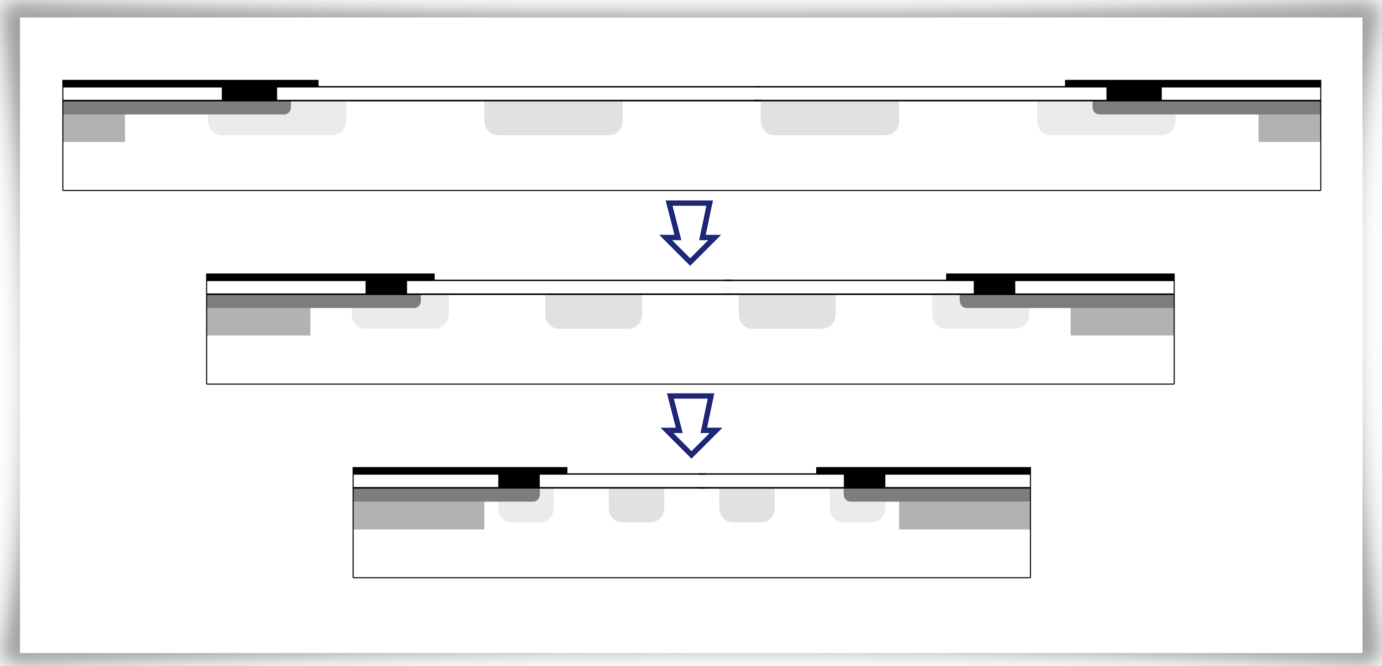
2. Analysis about the internal **electric fields**: efficiency issues, …



Design and simulation of LGAD

2. Analysis about the internal **electric fields**: efficiency issues, …

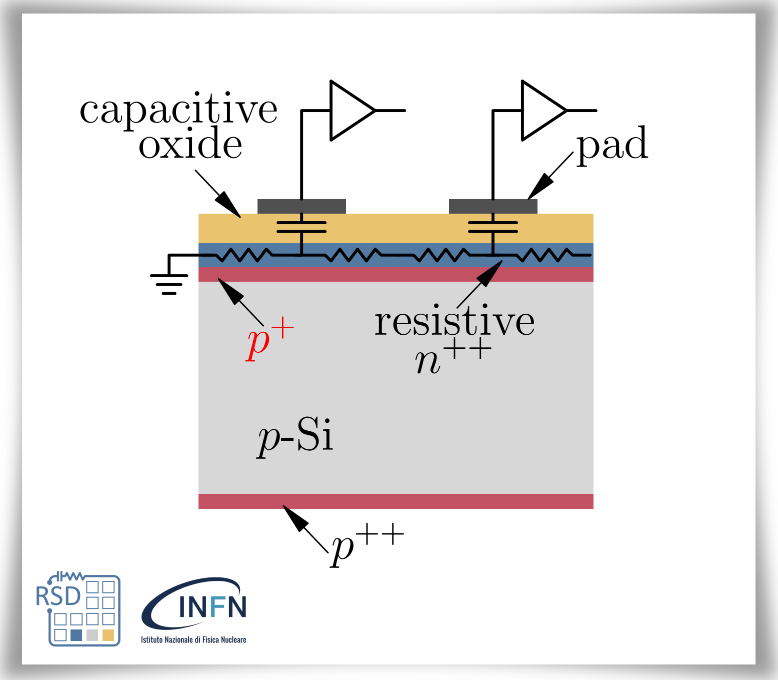
- **Two main strategies:**
  a. layout scaling

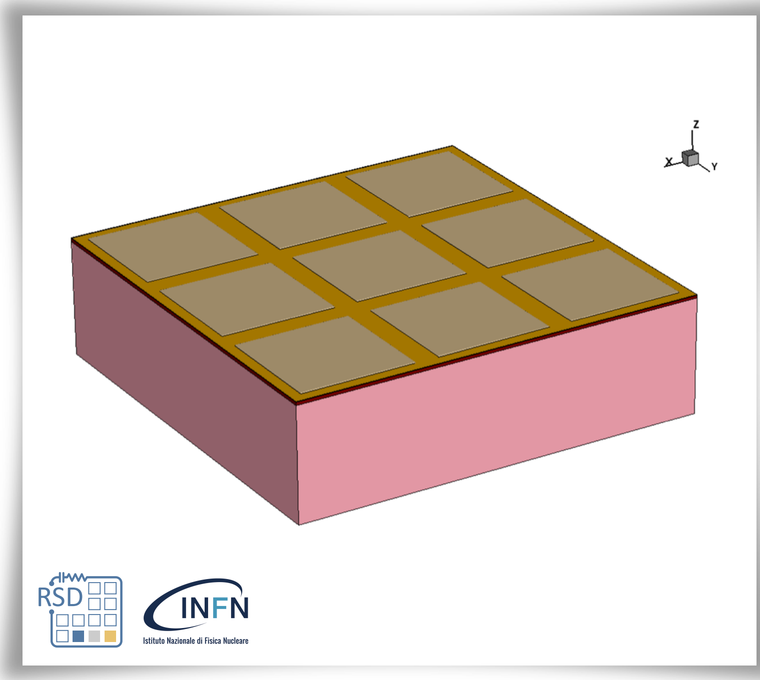


Design and simulation of LGAD

2. Analysis about the internal **electric fields**: efficiency issues, …

- **Two main strategies:**
  - *b. implement a new readout approach*





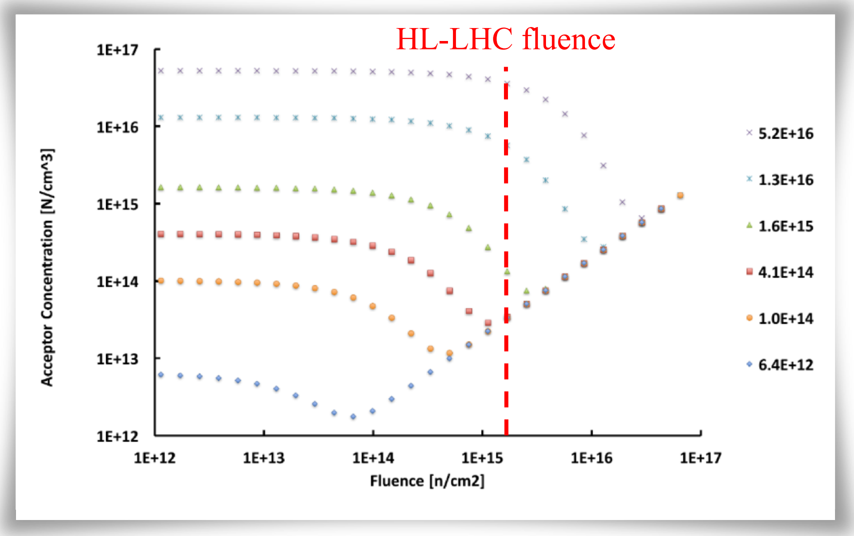
Design and simulation of LGAD

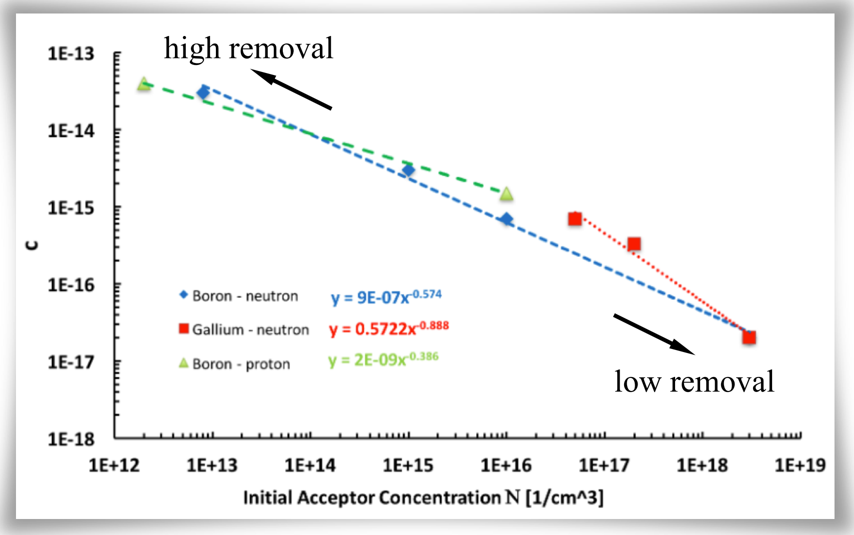
3. Radiation tolerance

empirical **acceptor removal/creation** law

$$N_A(\phi, x) = g_{\text{eff}}\,\phi + N_A(0, x)\,e^{-c(N_A(0,x))\phi}$$

$$c\,(N_A(0, x)) = \alpha\,N_A(0, x)^{-\beta}$$





Design and simulation of LGAD

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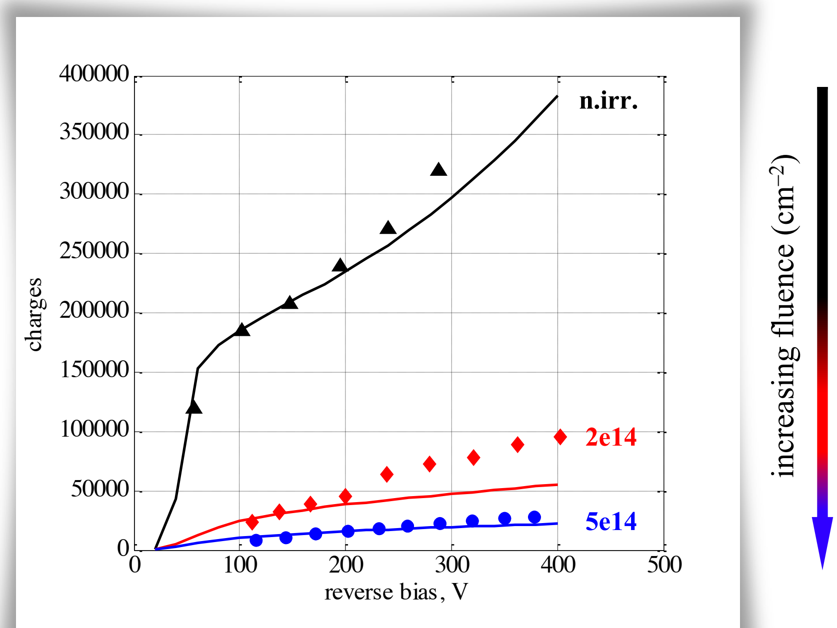


Design and simulation of LGAD

1. Study of detector **static/dynamic characteristics**: *I*(*V*), *G*(*V*), *C*(*V*), …

+

2. Analysis about the internal **electric fields**: efficiency issues, …

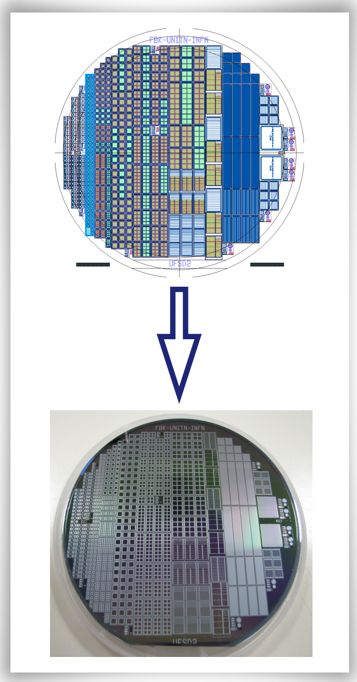
+

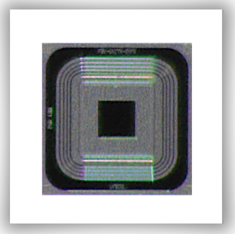
3. **Radiation tolerance**

1. + 3.

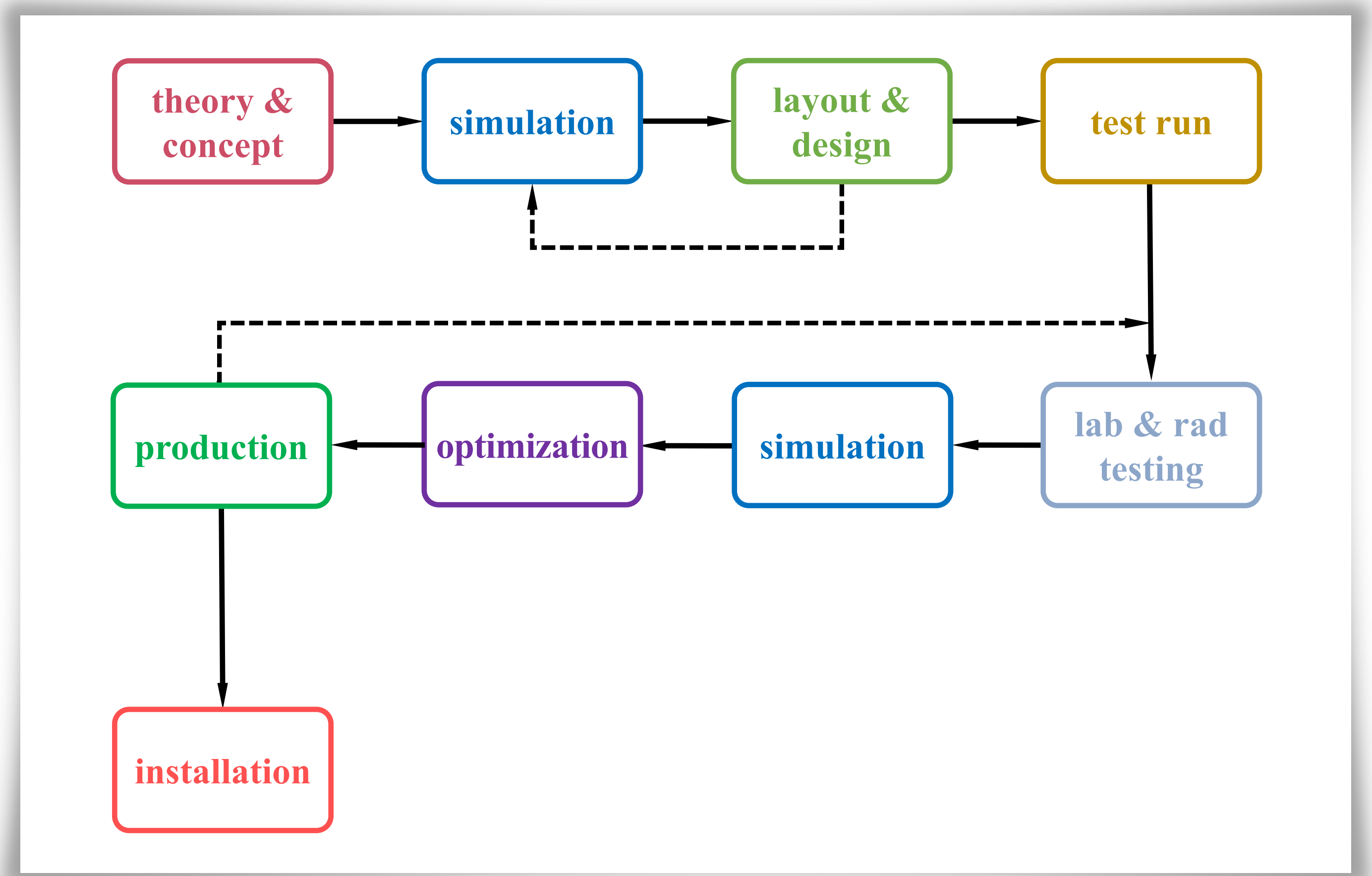
Wafer #	Dopant	Gain dose	Carbon
1	Boron	0.98	
2	Boron	1.00	
3	Boron	1.00	
4	Boron	1.00	low
5	Boron	1.00	High
6	Boron	1.02	low
7	Boron	1.02	High
8	Boron	1.02	
9	Boron	1.02	
10	Boron	1.04	
11	Gallium	1.00	
14	Gallium	1.04	
15	Gallium	1.04	low
16	Gallium	1.04	High
18	Gallium	1.08	

2.





LGAD production: the complete workflow!



Contacts and Info

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